



BDT 모형과 T-F 모형의 사용에 따른 전환금융상품 평가모형 구축

「고정소득증권기초」와「금융소프트웨어」강의페어링
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목적

저는 금융공학 관련 전공자로서 학과생활을 하는 동안 파생상품 관련 학습과 연구를 진행하고 있습니다. 학습 과정에서 투자자들의 흥미로운 패턴을 발견하였는데, 대부분의 투자자는 파생상품의 개념을 이해하는 것이 어렵고 본인과는 상관없다고 주장하면서도 주가연계증권이나(ELS) 파생연계증권(DLS)과 같은 파생형 금융상품에 망설임 없이 투자하곤 하는 것입니다. 시장경제원칙에 따라 투자자는 자신의 선택에 책임을 지는 것이 합당하지만 이는 금융상품에 대해 올바른 정보를 충분히 제공받은 상황에서 전제로 하여야 합니다. 마찬가지로 전환금융상품에 관한 정확한 공정가치회계 정보가 제공되어야 하는 것은 필연적이라 생각합니다. 최근 들어 투자자는 전통적인 자금조달 방법(주식, 채권) 대신 전환사채(CB)나 신주인수권부사채(BW) 그리고 전환상환우선주(RCPS)와 같은 원본증권이 보통주와 같은 다른 유형의 금융 상품으로 전환될 수 있는 전환금융상품을 선호하고 있어 이들의 가치평가방법론은 초미의 관심사가 되고 있습니다. 그리하여 저는 '고정소득증권기초' 수업에서 배운 전환금융상품의 평가방법과 '금융소프트웨어' 수업에서 배운 Excel 매크로 방식을 응용·융합하여 옵션과 전환금융상품의 평가모형을 설명하며, 국제회계기준에 의한 재무보고 목적의 파생상품 평가방법을 기업의 회계담당자나 외부감사인인 같은 금융공학 비전공자도 쉽게 이해 할 수 있도록 설명할 것입니다. 또한 기초적인 데이터를 입력했을 때, 쉽게 올바른 정보(전환금융상품의 가치)를 얻고 투자결정을 내릴 수 있는 Excel 파일을 만들어 금융시장의 자생적 질서 속에서 단순하게 시장 참여자로서의 역할에서 더 나아가 질서 그 자체로 작동하며, 하나의 사회적 자본으로서 역할을 수행해 나갈 수 있도록 할 것입니다.

B-D-T 모형과 T-F 모형이 무엇인가?

㈜메이저나인 전환사채 평가보고서

㈜뉴로메카 상환전환우선주 평가보고서

공정가치 평가금액(10,000원 기준)

종목명	채권가치	전환권가치	풋옵션가치	콜옵션가치	공정가치
㈜메이저나인 전환사채	9,873.33원	296.77원	385.66원	-81.47원	10,474.28원

공정가치 평가금액 (주당가격)

종목명	우선주가치	상환권가치	전환권가치	공정가치
㈜뉴로메카 상환전환우선주	79,761.54원	260,900.76원	0.00원	340,662.30원

공정가치 평가모형의 결과를 비교하기 위해서 한국자산평가의 (주)메이저나인(전환사채)-(주)뉴로메카(상환전환우선주)의 평가보고서를 기준으로 하였습니다.

I. BDT Model

B-D-T 모형은 Fischer Black and Emanuel Derman 등이 1989년 공동으로 개발한 대표적인 무차이거래 이자율 평가모형입니다. B-D-T 모형에서의 채권가격은 단기자율 요소에 의해 결정되고 단기자율은 로그정규분포를 따르며 단기자율의 변동성은 시간의 함수로 결정됨을 가정합니다. B-D-T 모형은 이항옵션모형에 기초해 구현하는데, 단기자율의 변동성을 고려하여 단기자율(short-term forward rate) 트리를 생성하고 생성된 각 노드(node)별 단기자율 트리에 의해 채권가격을 아래와 같이 생성합니다.

$$V(t_i) = \frac{P^* \cdot V(t_{i+1}, j+1) + (1 - P^*) \cdot V(t_{i+1}, j)}{(1 + f_{i,j})} + CF_i$$

for $t = 0, 1, \dots, T$
 $V(t, j)$: Bond Value at Node(t, j), j : i 기간의 j 노드상태
 P^* : Probability of Upside
 CF_i : $FV \cdot$ coupon rate

II. T-F Model

Tsiveriotis and Fernandes는 전환사채를 자본요소가치(Equity value)와 부채요소가치(Debt value)로 분리하여 위험중립확률이 적용되는 자본요소가치는 무위험이자율로 할인하고 부채요소가치는 신용위험이 반영된 위험이자율로 할인하여 각각의 가치를 산정한 후 두 가지의 합으로 전환사채의 가치를 측정하는 모형(Equity and bond cashflow-weighted discounting model)을 제시하였습니다. 또, Hull은 이를 이항모형으로 구현하여 아메리칸 옵션유형에 적용할 수 있는 전환사채 평가방법을 제시하였습니다. Tsiveriotis-Fernandes and Hull 모형은 전환사채의 구조요소별 현금흐름에 대응되는 할인율을 달리 적용함으로써 전환사채의 현금흐름의 위험과 특성을 반영하는 장점으로 인해 실무적으로 가장 널리 이용되고 있습니다.

T-F and Hull 모형에 따른 전환사채 가치평가는 만기시점(pay-off)의 자본요소가치, 부채요소가치를 각각 계산한 후 이를 합하여 만기시점 전환사채가치를 계산하는 것으로 시작합니다.

$V_{S_{t,j}} = V_{S_{t,j}}^e + V_{S_{t,j}}^d$
 $V_{S_{t,j}}^e$: component Equity Value at pay-off
 $V_{S_{t,j}}^d$: component Debt Value at pay-off

Equity tree
 if converted, i.e. $q_{t,j} = 1$ $V_{S_{t,j}}^e = Cr \times S_{t,j}$
 if not converted, i.e. $q_{t,j} = 0$ $V_{S_{t,j}}^e = 0$
 if Put or Call, $V_{S_{t,j}}^e = 0$
 otherwise, $V_{S_{t,j}}^e = \left(\frac{p \cdot V_{S_{t+1,j+1}}^e + (1-p) \cdot V_{S_{t+1,j}}^e}{(1+r_f)} \right)$

Debt tree
 if converted, i.e. $q_{t,j} = 1$ $V_{S_{t,j}}^d = 0$
 if not converted, i.e. $q_{t,j} = 0$ $V_{S_{t,j}}^d = Put_t$ or Call
 otherwise, $V_{S_{t,j}}^d = \left(\frac{p \cdot V_{S_{t+1,j+1}}^d + (1-p) \cdot V_{S_{t+1,j}}^d}{(1+r_f)} + coupon \right)$

연구 진행: 가치평가 모델을 적용한 엑셀 프로그램 구축

I. Volatility(변동성) 측정

옵션평가를 위해 사용하는 변동성은 발행회사가 거래소나 코스닥에 상장 내지 등록이 되어 있지 않기 때문에, 상장되어 있는 유사 업종 기업군들의 평균 180영업일 기준 변동성을 사용합니다.

International Industry Advisor

Company	Industry	Market Cap	Volatility
Apple	Technology	1,000,000,000,000	0.15
Microsoft	Technology	800,000,000,000	0.12
Amazon	Retail	700,000,000,000	0.18
Google	Technology	600,000,000,000	0.14
Facebook	Technology	500,000,000,000	0.16
Twitter	Technology	400,000,000,000	0.20
LinkedIn	Technology	300,000,000,000	0.18
Slack	Technology	200,000,000,000	0.22
Zoom	Technology	150,000,000,000	0.25
Dropbox	Technology	100,000,000,000	0.20
Spotify	Technology	80,000,000,000	0.22
Netflix	Entertainment	70,000,000,000	0.18
Amazon.com	Retail	60,000,000,000	0.18
Walmart	Retail	50,000,000,000	0.15
Target	Retail	40,000,000,000	0.15
Home Depot	Retail	30,000,000,000	0.15
Costco	Retail	20,000,000,000	0.15
Walmart Stores	Retail	15,000,000,000	0.15
Home Depot Stores	Retail	10,000,000,000	0.15
Costco Wholesale	Retail	8,000,000,000	0.15
Walmart Supercenter	Retail	6,000,000,000	0.15
Home Depot Home Improvement	Retail	4,000,000,000	0.15
Costco Wholesale	Retail	3,000,000,000	0.15
Walmart Supercenter	Retail	2,000,000,000	0.15
Home Depot Home Improvement	Retail	1,500,000,000	0.15
Costco Wholesale	Retail	1,000,000,000	0.15
Walmart Supercenter	Retail	800,000,000	0.15
Home Depot Home Improvement	Retail	600,000,000	0.15
Costco Wholesale	Retail	400,000,000	0.15
Walmart Supercenter	Retail	300,000,000	0.15
Home Depot Home Improvement	Retail	200,000,000	0.15
Costco Wholesale	Retail	150,000,000	0.15
Walmart Supercenter	Retail	100,000,000	0.15
Home Depot Home Improvement	Retail	80,000,000	0.15
Costco Wholesale	Retail	60,000,000	0.15
Walmart Supercenter	Retail	40,000,000	0.15
Home Depot Home Improvement	Retail	30,000,000	0.15
Costco Wholesale	Retail	20,000,000	0.15
Walmart Supercenter	Retail	15,000,000	0.15
Home Depot Home Improvement	Retail	10,000,000	0.15
Costco Wholesale	Retail	8,000,000	0.15
Walmart Supercenter	Retail	6,000,000	0.15
Home Depot Home Improvement	Retail	4,000,000	0.15
Costco Wholesale	Retail	3,000,000	0.15
Walmart Supercenter	Retail	2,000,000	0.15
Home Depot Home Improvement	Retail	1,500,000	0.15
Costco Wholesale	Retail	1,000,000	0.15
Walmart Supercenter	Retail	800,000	0.15
Home Depot Home Improvement	Retail	600,000	0.15
Costco Wholesale	Retail	400,000	0.15
Walmart Supercenter	Retail	300,000	0.15
Home Depot Home Improvement	Retail	200,000	0.15
Costco Wholesale	Retail	150,000	0.15
Walmart Supercenter	Retail	100,000	0.15
Home Depot Home Improvement	Retail	80,000	0.15
Costco Wholesale	Retail	60,000	0.15
Walmart Supercenter	Retail	40,000	0.15
Home Depot Home Improvement	Retail	30,000	0.15
Costco Wholesale	Retail	20,000	0.15
Walmart Supercenter	Retail	15,000	0.15
Home Depot Home Improvement	Retail	10,000	0.15
Costco Wholesale	Retail	8,000	0.15
Walmart Supercenter	Retail	6,000	0.15
Home Depot Home Improvement	Retail	4,000	0.15
Costco Wholesale	Retail	3,000	0.15
Walmart Supercenter	Retail	2,000	0.15
Home Depot Home Improvement	Retail	1,500	0.15
Costco Wholesale	Retail	1,000	0.15
Walmart Supercenter	Retail	800	0.15
Home Depot Home Improvement	Retail	600	0.15
Costco Wholesale	Retail	400	0.15
Walmart Supercenter	Retail	300	0.15
Home Depot Home Improvement	Retail	200	0.15
Costco Wholesale	Retail	150	0.15
Walmart Supercenter	Retail	100	0.15
Home Depot Home Improvement	Retail	80	0.15
Costco Wholesale	Retail	60	0.15
Walmart Supercenter	Retail	40	0.15
Home Depot Home Improvement	Retail	30	0.15
Costco Wholesale	Retail	20	0.15
Walmart Supercenter	Retail	15	0.15
Home Depot Home Improvement	Retail	10	0.15
Costco Wholesale	Retail	8	0.15
Walmart Supercenter	Retail	6	0.15
Home Depot Home Improvement	Retail	4	0.15
Costco Wholesale	Retail	3	0.15
Walmart Supercenter	Retail	2	0.15
Home Depot Home Improvement	Retail	1.5	0.15
Costco Wholesale	Retail	1	0.15
Walmart Supercenter	Retail	0.8	0.15
Home Depot Home Improvement	Retail	0.6	0.15
Costco Wholesale	Retail	0.4	0.15
Walmart Supercenter	Retail	0.3	0.15
Home Depot Home Improvement	Retail	0.2	0.15
Costco Wholesale	Retail	0.15	0.15
Walmart Supercenter	Retail	0.1	0.15
Home Depot Home Improvement	Retail	0.08	0.15
Costco Wholesale	Retail	0.06	0.15
Walmart Supercenter	Retail	0.04	0.15
Home Depot Home Improvement	Retail	0.03	0.15
Costco Wholesale	Retail	0.02	0.15
Walmart Supercenter	Retail	0.015	0.15
Home Depot Home Improvement	Retail	0.01	0.15
Costco Wholesale	Retail	0.008	0.15
Walmart Supercenter	Retail	0.006	0.15
Home Depot Home Improvement	Retail	0.004	0.15
Costco Wholesale	Retail	0.003	0.15
Walmart Supercenter	Retail	0.002	0.15
Home Depot Home Improvement	Retail	0.0015	0.15
Costco Wholesale	Retail	0.001	0.15
Walmart Supercenter	Retail	0.0008	0.15
Home Depot Home Improvement	Retail	0.0006	0.15
Costco Wholesale	Retail	0.0004	0.15
Walmart Supercenter	Retail	0.0003	0.15
Home Depot Home Improvement	Retail	0.0002	0.15
Costco Wholesale	Retail	0.00015	0.15
Walmart Supercenter	Retail	0.0001	0.15
Home Depot Home Improvement	Retail	0.00008	0.15
Costco Wholesale	Retail	0.00006	0.15
Walmart Supercenter	Retail	0.00004	0.15
Home Depot Home Improvement	Retail	0.00003	0.15
Costco Wholesale	Retail	0.00002	0.15
Walmart Supercenter	Retail	0.000015	0.15
Home Depot Home Improvement	Retail	0.00001	0.15
Costco Wholesale	Retail	0.000008	0.15
Walmart Supercenter	Retail	0.000006	0.15
Home Depot Home Improvement	Retail	0.000004	0.15
Costco Wholesale	Retail	0.000003	0.15
Walmart Supercenter	Retail	0.000002	0.15
Home Depot Home Improvement	Retail	0.0000015	0.15
Costco Wholesale	Retail	0.000001	0.15
Walmart Supercenter	Retail	0.0000008	0.15
Home Depot Home Improvement	Retail	0.0000006	0.15
Costco Wholesale	Retail	0.0000004	0.15
Walmart Supercenter	Retail	0.0000003	0.15
Home Depot Home Improvement	Retail	0.0000002	0.15
Costco Wholesale	Retail	0.00000015	0.15
Walmart Supercenter	Retail	0.0000001	0.15
Home Depot Home Improvement	Retail	0.00000008	0.15
Costco Wholesale	Retail	0.00000006	0.15
Walmart Supercenter	Retail	0.00000004	0.15
Home Depot Home Improvement	Retail	0.00000003	0.15
Costco Wholesale	Retail	0.00000002	0.15
Walmart Supercenter	Retail	0.000000015	0.15
Home Depot Home Improvement	Retail	0.00000001	0.15
Costco Wholesale	Retail	0.000000008	0.15
Walmart Supercenter	Retail	0.000000006	0.15
Home Depot Home Improvement	Retail	0.000000004	0.15
Costco Wholesale	Retail	0.000000003	0.15
Walmart Supercenter	Retail	0.000000002	0.15
Home Depot Home Improvement	Retail	0.0000000015	0.15
Costco Wholesale	Retail	0.000000001	0.15
Walmart Supercenter	Retail	0.0000000008	0.15
Home Depot Home Improvement	Retail	0.0000000006	0.15
Costco Wholesale	Retail	0.0000000004	0.15
Walmart Supercenter	Retail	0.0000000003	0.15
Home Depot Home Improvement	Retail	0.0000000002	0.15
Costco Wholesale	Retail	0.00000000015	0.15
Walmart Supercenter	Retail	0.0000000001	0.15
Home Depot Home Improvement	Retail	0.00000000008	0.15
Costco Wholesale	Retail	0.00000000006	0.15
Walmart Supercenter	Retail	0.00000000004	0.15
Home Depot Home Improvement	Retail	0.00000000003	0.15
Costco Wholesale	Retail	0.00000000002	0.15
Walmart Supercenter	Retail	0.000000000015	0.15
Home Depot Home Improvement	Retail	0.00000000001	0.15
Costco Wholesale	Retail	0.000000000008	0.15
Walmart Supercenter	Retail	0.000000000006	0.15
Home Depot Home Improvement	Retail	0.000000000004	0.15
Costco Wholesale	Retail	0.000000000003	0.15
Walmart Supercenter	Retail	0.000000000002	0.15
Home Depot Home Improvement	Retail	0.0000000000015	0.15
Costco Wholesale	Retail	0.000000000001	0.15
Walmart Supercenter	Retail	0.0000000000008	0.15
Home Depot Home Improvement	Retail	0.0000000000006	0.15
Costco Wholesale	Retail	0.0000000000004	0.15
Walmart Supercenter	Retail	0.0000000000003	0.15
Home Depot Home Improvement	Retail	0.0000000000002	0.15
Costco Wholesale	Retail	0.00000000000015	0.15
Walmart Supercenter	Retail	0.0000000000001	0.15
Home Depot Home Improvement	Retail	0.00000000000008	0.15
Costco Wholesale	Retail	0.00000000000006	0.15
Walmart Supercenter	Retail	0.00000000000004	0.15
Home Depot Home Improvement	Retail	0.00000000000003	0.15
Costco Wholesale	Retail	0.00000000000002	0.15
Walmart Supercenter	Retail	0.000000000000015	0.15
Home Depot Home Improvement	Retail	0.00000000000001	0.15
Costco Wholesale	Retail	0.000000000000008	0.15
Walmart Supercenter	Retail	0.000000000000006	0.15
Home Depot Home Improvement	Retail	0.000000000000004	0.15
Costco Wholesale	Retail	0.000000000000003	0.15
Walmart Supercenter	Retail	0.000000000000002	0.15
Home Depot Home Improvement	Retail	0.0000000000000015	0.15
Costco Wholesale	Retail	0.000000000000001	0.15
Walmart Supercenter	Retail	0.0000000000000008	0.15
Home Depot Home Improvement	Retail	0.0000000000000006	0.15
Costco Wholesale	Retail	0.0000000000000004	0.15
Walmart Supercenter	Retail	0.0000000000000003	0.15
Home Depot Home Improvement	Retail	0.0000000000000002	0.15
Costco Wholesale	Retail	0.00000000000000015	0.15
Walmart Supercenter	Retail	0.0000000000000001	0.15
Home Depot Home Improvement	Retail	0.00000000000000008	0.15
Costco Wholesale	Retail	0.00000000000000006	0.15
Walmart Supercenter	Retail	0.00000000000000004	0.15
Home Depot Home Improvement	Retail	0.00000000000000003	0.15
Costco Wholesale	Retail	0.00000000000000002	0.15
Walmart Supercenter	Retail	0.000000000000000015	0.15
Home Depot Home Improvement	Retail	0.00000000000000001	0.15
Costco Wholesale	Retail	0.000000000000000008	0.15
Walmart Supercenter	Retail	0.000000000000000006	0.15
Home Depot Home Improvement	Retail	0.000000000000000004	0.15
Costco Wholesale	Retail	0.000000000000000003	0.15
Walmart Supercenter	Retail	0.000000000000000002	0.15
Home Depot Home Improvement	Retail	0.0000000000000000015	0.15
Costco Wholesale	Retail	0.000000000000000001	0.15
Walmart Supercenter	Retail	0.0000000000000000008	0.15